

PR-SQUARED: A NEW DESCRIPTION

PR-Squared is a new electoral system designed for the UK's House of Commons. It typically elects a majority government; it elects one local MP from each constituency each of whom is dependent on the local vote; yet it still ensures that equal votes mean equal seats.

PR-Squared works as follows:

- The country is divided into a large number of single-member constituencies;
- In each constituency each party may field one candidate;
- Voters cast a single vote in favour of a single candidate;
- The votes for each party are totalled nation-wide;
- The key rule: each party is allocated seats in proportion to the square of its nation-wide vote;
- As only a whole number of seats can be won, the seat allocations must be rounded, which is done using the method of major fractions¹.
- It is now known how many seats each party has won, but not which constituencies. **Constituencies are allocated to the parties in the manner that maximises the nation-wide total of the number of voters who voted for their local MP.** Equivalently, define a 'happy voter' to be a voter who voted for his or her MP, and then assign seat winners so as to maximise the nation's total 'happiness'. In practice this will be First-Past-The-Post (FPTP) in non-marginal seats, with marginal seats being rearranged to ensure that parties receive the required number of MPs.

We start with a simple example with only three parties and seven constituencies, in which votes are as follows:

Constituencies:	Palatine	Capitoline	Aventine	Cælian	Esquiline	Viminal	Quirinal	Totals
Red	6	5	5	3	2	4	3	28
Blue	4	3	2	5	5	0	1	20
Yellow	0	0	2	1	1	6	4	14
Totals	10	8	9	9	8	10	8	62

The number of seats each party won is calculated from the parties' nation-wide vote totals: 28, 20 and 14. The seven seats are allocated proportional to the squares of these, giving an unrounded allocation of 3.98, 2.03 and 0.99, and hence a rounded allocation of 4, 2 and 1.

But which party has won which seat? Let's guess. If the first four seats were allocated to Red (Palatine, Capitoline, Aventine and Cælian), the next two to Blue (Esquiline and Viminal), and the last to yellow (Quirinal), then 28 voters across the nation would have voted for their MP. We say that, under this seat assignment, 28 voters are 'happy'. PR-Squared allocates seats by maximising happiness. A computerised algorithm quickly shows that the maximum happiness is 35: Red wins Palatine, Capitoline, Aventine and Quirinal, Blue takes Cælian and Esquiline and Yellow Viminal.

¹ Also known as the method of odd numbers, Webster's method, and the method of Saint-Laguë. The original definition of PR-Squared rounded by the Largest Remainder rule: the difference between the two will only very rarely be more than one seat for any party.

We bring this into the table, along with the First-Past-The-Post winner:

Constituencies:	Palatine	Capitoline	Aventine	Caelian	Esquiline	Viminal	Quirinal	Totals
Red	6	5	5	3	2	4	3	28
Blue	4	3	2	5	5	0	1	20
Yellow	0	0	2	1	1	6	4	14
Totals	10	8	9	9	8	10	8	62
FPTP winner:	Red	Red	Red	Blue	Blue	Yellow	Yellow	3:2:2
PR-Squared w.:	Red	Red	Red	Blue	Blue	Yellow	Red	4:2:1

Observe that:

- In six out of the seven seats, the PR-Squared winner is the same as the FPTP winner. In practice, PR-Squared assigns the non-marginal constituencies to the FPTP winner; but reassigns some of the marginal constituencies to make the national totals work. This does not diminish the importance of a vote cast in a marginal constituency: governments are chosen by nation-wide vote totals, and hence all votes are equal, whether cast in a deep-red, a deep-blue, or a marginal constituency.
- MPs are dependent on the local vote. If a single voter in Quirinal had switched from Red to Yellow, then Red's Quirinal candidate would have lost to Yellow's Quirinal candidate. Such a switch would have made Quirinal a less-marginal constituency, so Yellow would hold it and would instead lose Viminal (one voter switching isn't enough to change the seat totals; if three switched from Red to Yellow, then Reds would win one seat fewer, and Yellows one more). Hence MPs are truly local; a candidate requires a local vote, and the voters have the power to dismiss an individual MP.
- There is no need for the constituencies to be of equal size. Because governments are chosen by nation-wide vote totals, the fairness of PR-Squared is not reduced by having unequal constituency sizes. Further, there is no purpose to gerrymandering.

Let's consider an actual election result. In the UK 1983 general election the vote for the three large parties split the proportion 44.5% to 28.9% to 26.6%. Seats would have been allocated in proportion to the squares of these numbers: 1980.25, 835.21 and 707.56. Scaling the ratio of the squares so that they total 650 seats gives 365.4, 154.1 and 130.5, which round to an actual seat allocation of 365, 154 and 131: a majority of 80 for the largest party². Note that equal votes give equal seats, and nearly equal votes give nearly equal seats (unlike the first-past-the-post, in which the second-largest party received 1.09 times as many votes as the third largest, but 9 times as many seats).

² We counter-factually assume that the vote totals would be the same under PR-Squared as under FPTP, as such an assumption is near enough to be true for these purposes. However, there is one particular difficulty with mapping FPTP results onto PR-Squared: under FPTP, not all parties stand in all seats. This is because the parties have no incentive to stand where there is no chance of winning. But for completeness PR-Squared needs a rule to cover the case in which, in a particular constituency, no seat-winning party put forward a candidate (i.e., all the candidates are from small local parties). However, whilst there ought to be a rule, in practice, the parties' incentive is such that this rule would never be needed. Parties want votes, and any party with a chance of winning seats anywhere will want every vote it can get in any constituency. But there should be a rule nonetheless, and it would work as follows. In each constituency in which a party did not put forward a candidate, it is assumed that the party did put forward a candidate (a 'deemed null candidate'), but that this candidate received zero votes; and any seat won by a deemed null candidate would remain empty.

As a further example, the following table shows the 1997 UK election in detail:

Party	Votes	Votes Squared	Unrounded seats	Actual Seats
Labour	12,941,957	167,494bn	389.24	389
Conservative	9,600,940	92,178bn	214.21	214
Liberal Democrat	4,724,626	22,322bn	51.87	52
Referendum Party	811,679	659bn	1.53	2
Scottish National Party	617,260	381bn	0.89	1
Labour Co-operative	599,423	359bn	0.83	1
Ulster Unionist Party	258,349	67bn	0.16	0
S. D. & L. P.	192,060	37bn	0.09	0
Plaid Cymru	161,030	26bn	0.06	0
Sinn Fein	126,921	16bn	0.04	0
D. U. P.	107,348	12bn	0.03	0
UK Independence Party	106,001	11bn	0.03	0
others	small	small	small	0

(counting Miss Betty Boothroyd, the Speaker, for Labour)

And who would have won which seat? As a randomly chosen example, in Sedgefield the Labour Party candidate received 33526 votes, against 8383 for the (second-placed) Conservative Party candidate. This would have been sufficient to ensure that the happiness maximisation allocated this seat to the Labour candidate. Indeed, this seat would have still been won by Labour if fewer than 23710 of those who had voted Labour had stayed in bed that day: the winning MP's majority could be said to be 23710.

By-elections are slightly more awkward under PR-squared than under FPTP. If a seat should become empty, there are two possibilities:

- The first possibility is that by-elections don't happen. Empty seats would remain empty until the next general election.
- The second possibility is that a FPTP-style by-election is held.

This second possibility requires care. PR-Squared gives parties an incentive to negotiate coalitions before an election, and to court votes without regard to geography; it gives candidates an incentive to encourage both the local and the national votes; and gives voters an incentive to vote for a party with a realistic chance of forming a government. Care must be taken to ensure that the rules about by-elections do not create any unwanted new incentives.

In the UK, an armed terrorist and racketeering organisation has a political front. Such support as it has is geographically narrow, and under FPTP it wins only a small number of constituencies (usually two, in 2001 four). Under PR-Squared, it is highly unlikely that any seats would be won by the terrorists' front. But in certain constituencies it might win a local by-election, which gives the terrorists an incentive to 'cause' a by-election. This is not an idle concern, as both bombings and assassinations have continued during their 'cease-fire'. So death by murder should not cause a by-election. But if resignations cause a by-election, then the terrorists would have an incentive to kill the sitting MP's children one by one, until the MP is persuaded to resign. These are not incentives that an electoral system should be creating. So, if there are to be FPTP-style by-elections, they should be triggered only by the death from natural causes of a sitting MP³.

³ This also raises the issue of the death of a candidate just before an election. What should happen if a candidate dies after being nominated but before the polling date? To withdraw the candidate from the poll denies voters the opportunity to vote for that party, which seems unfair. The natural course is to allow the dead candidate to remain a candidate: for electoral purposes the candidate will be deemed to have died immediately after the election. If the dead candidate is elected, the by-election rules then come into force.

Hence PR-Squared has many advantages.

- PR-Squared elects a government, rather than a collection of parties who then negotiate for power. If parties are sufficiently close in policies to be likely to form a coalition, then they have a strong incentive to do so before rather than after the election, and thus to do so in a manner visible to the electorate. For example, if five parties split the vote equally, then the seats would split equally. But if two of these parties could form a coalition before the election, and still hold 40% of the vote, then that coalition would win four-sevenths of the seats, a functional majority. Thus PR-Squared keeps power at the ballot box rather than at a negotiating table.
- Likewise, voters have an incentive to vote for a large party that has a plausible chance of forming a government, rather than a no-hope extremist party.
- But every MP is still a local constituency MP, dependent on the local vote. An unpopular or corrupt MP could lose, even if that MP came from a large party.
- PR-Squared's voting mechanism is very simple: put a single cross by a single candidate. There would not be any need for parties to distribute 'voting sheets' that show how to order complicated lists of preferences.
- PR-Squared extends voter choice, at least relative to the UK's current electoral system. In Liverpool there is little purpose in voting under FPTP; the Labour vote is so overwhelming that it could be weighed rather than counted. But under PR-Squared, voting is still important. An additional Labour vote won't make any difference in Liverpool, but it might make the difference somewhere else, as might a vote for the Conservatives or the LibDems. However, whilst there is an extension of voter choice, it is a limited extension: a vote for the communists or the fascists would still be a wasted vote.

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