In[1]:=
```
Quiet[Remove["Global`*"], {Remove::rmnsm}];
Print["Mathematica $Version = “", $Version, "”"];
Print["Execution time = ", DateString[DateList[], {"Hour", ":", "Minute", " on ",
   "DayNameShort", " ", "Day", " ", "MonthNameShort", " ", "Year"}]];
```

```
Mathematica $Version = “9.0 for Mac OS X x86 (64-bit) (January 24, 2013)”

Execution time = 01:03 on Thu 11 Jul 2019
```

In[2]:=
```
(* potential function p[x,y] ≡ p. *)
(* pX ≡ δp/δx;   pY ≡ δp/δy;  pXX ≡ δ²p/δx²;
pYY ≡ δ²p/δy²;  pXY ≡ δ²p/δXδy *)
(* If walking uphill along Grad[p]
 how fast does one turn as angle per distance. *)
```

In[3]:=
```
chgAng =
  (ArcTan[(pY + eps (pYY pY + pXY pX)) / Sqrt[pX^2 + pY^2]] /
     (pX + eps (pXX pX + pXY pY)) / Sqrt[pX^2 + pY^2]] - ArcTan[pY/pX]) // FullSimplify
```

Out[3]=
$$-\text{ArcTan}\left[\frac{pY}{pX}\right]+\text{ArcTan}\left[\frac{\text{eps}\ pX\ pXY+pY\sqrt{pX^2+pY^2}+\text{eps}\ pY\ pYY}{\text{eps}\ pX\ pXX+\text{eps}\ pXY\ pY+pX\sqrt{pX^2+pY^2}}\right]$$

In[4]:=
```
dAngleBydDistance = Limit[chgAng/eps, eps → 0] // FullSimplify
```

Out[4]=
$$\frac{pX^2\ pXY-pXY\ pY^2+pX\ pY\ (-pXX+pYY)}{\left(pX^2+pY^2\right)^{3/2}}$$

In[5]:=
```
(* Units are distance^-4/distance^-3, which is lightly reassuring. *)
```

In[6]:=

```
(* And a check *)
```

In[7]:=
```
(D[chgAng, eps] /. {eps → 0}) // FullSimplify
```

Out[7]=
$$\frac{pX^2\ pXY-pXY\ pY^2+pX\ pY\ (-pXX+pYY)}{\left(pX^2+pY^2\right)^{3/2}}$$