```
(* www.jdawiseman.com/2016/20160420_Arc_Radius_Droog.nb *)
(* www.jdawiseman.com/2016/20160420_Arc_Radius_Droog.pdf *)
(* www.jdawiseman.com/author.html *)

Quiet[Remove["Global`*"], {Remove::rmnsm}];
Print["Mathematica $Version = ", $Version, "”"];
Print["Execution time = ", DateString[DateList[], {"Hour", ":", "Minute", " on ",
    "DayNameShort", " ", "Day", " ", "MonthNameShort", " ", "Year"}]];
```

Mathematica $Version = "9.0 for Mac OS X x86 (64-bit) (January 24, 2013)"

Execution time = 13:24 on Wed 20 Apr 2016

```
KnotsFromCoeffs[{c0_, c1_, c2_, c3_}] =
  {c0, c0 + c1 / 3, c0 + (2 c1 + c2) / 3, c0 + c1 + c2 + c3};
CoeffsFromKnots[{z0_, z1_, z2_, z3_}] =
  {z0, -3 z0 + 3 z1, 3 z0 - 6 z1 + 3 z2, -z0 + 3 z1 - 3 z2 + z3};

(* Simple checks *)
And[
    {z0, z1, z2, z3} == KnotsFromCoeffs[CoeffsFromKnots[{z0, z1, z2, z3}]],
    {c0, c1, c2, c3} == CoeffsFromKnots[KnotsFromCoeffs[{c0, c1, c2, c3}]]
] // Expand
```

True

```
(* https://groups.google.com/forum/#!topic/comp.lang.postscript/YiuM6lj5ngY *)

xAdobe = Table[t^i, {i, 0, 3}].CoeffsFromKnots[{1, 1, 552 / 1000, 0}];
yAdobe = Table[t^i, {i, 0, 3}].CoeffsFromKnots[{0, 552 / 1000, 1, 1}];
rSquaredAdobe = (xAdobe^2 + yAdobe^2) // FullSimplify
```

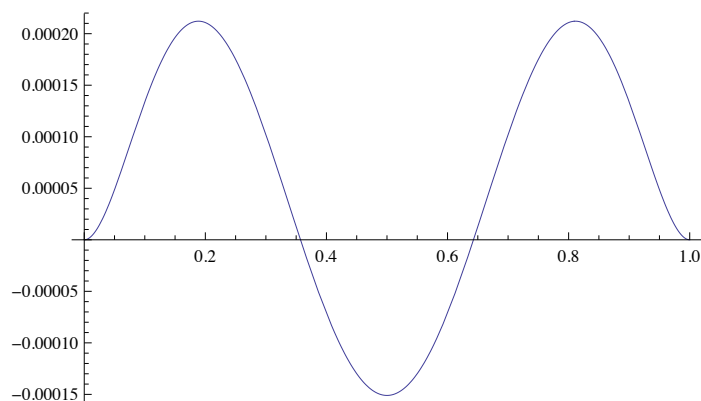$$\frac{15\,625 + (-1 + t)^2 \, t^2 \, (849 + 3698 \, (-1 + t) \, t)}{15\,625}$$

```
Sqrt[rSquaredAdobe] /. {t → 1 / 2}
```

$$\frac{707}{500 \sqrt{2}}$$

```
N[Sqrt[rSquaredAdobe /. {t → 1 / 2}], 25]
```

0.9998489885977781995027939

```
Plot[Sqrt[rSquaredAdobe] - 1, {t, 0, 1}]
```
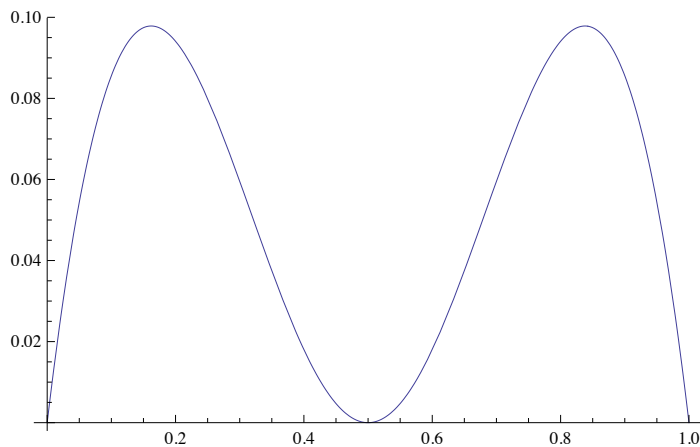
```
{xDroog, yDroog} = {
    Table[t^i, {i, 0, 3}].CoeffsFromKnots[{x0, x1, x2, x3}],
    Table[t^i, {i, 0, 3}].CoeffsFromKnots[{y0, y1, y2, y3}]
} //. {
    x0 → Cos[halfA],
    y0 → Sin[halfA],
    x1 → (4 - Cos[halfA]) / 3,
    y1 → ((3 - Cos[halfA]) (1 - Cos[halfA])) / Sin[halfA],
    x2 → x1, y2 → -y1,
    x3 → x0 , y3 → -y0
};
rSquaredDroog = (xDroog^2 + yDroog^2) /. {halfA → π / 4} // Simplify
```

$$1 + \left(14 - 8\sqrt{2}\right) t + \left(274 - 200\sqrt{2}\right) t^2 + 32\left(-61 + 43\sqrt{2}\right) t^3 +$$
$$\left(4416 - 3088\sqrt{2}\right) t^4 + 96\left(-43 + 30\sqrt{2}\right) t^5 - 32\left(-43 + 30\sqrt{2}\right) t^6$$

```
Plot[Sqrt[rSquaredDroog] - 1, {t, 0, 1}]
```



```
(* The above chart shows that the latest incarnation
   of the Droog formula is wrong. Very wrong. So let's help. *)
```

```
{xMiddle, yMiddle} = ({
    Table[t^i, {i, 0, 3}].CoeffsFromKnots[{x0, x1, x2, x3}],
    Table[t^i, {i, 0, 3}].CoeffsFromKnots[{y0, y1, y2, y3}]
} //. {
    z → Tan[a / 4] × 4 / 3,
    x0 → Cos[a / 2],
    y0 → Sin[a / 2],
    x1 → x0 + z × Sin[a / 2],
    y1 → y0 - z × Cos[a / 2],  (* Droog erroneously has a '+' sign here *)
    x2 → x1, y2 → -y1,
    x3 → x0 , y3 → -y0
}) // Simplify;
rSquaredMiddle = (xMiddle^2 + yMiddle^2) /. {a → π / 2} // FullSimplify
```

$$1 - 2\left(-17 + 12\sqrt{2}\right) t^2 \left(1 - 3t + 2t^2\right)^2$$

Plot[{Sqrt[rSquaredAdobe] - 1, Sqrt[rSquaredMiddle] - 1}, {t, 0, 1}]
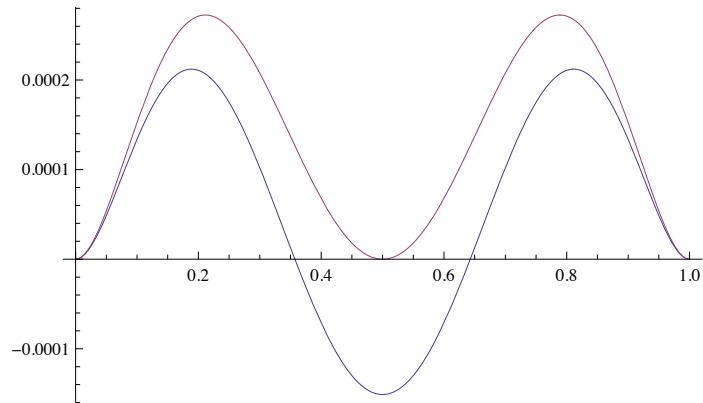
Solve[0 == D[rSquaredMiddle /. {a → π / 2}, t], t]

$$\left\{ \{t \to 0\}, \left\{t \to \frac{1}{2}\right\}, \{t \to 1\}, \left\{t \to \frac{1}{6}\left(3 - \sqrt{3}\right)\right\}, \left\{t \to \frac{1}{6}\left(3 + \sqrt{3}\right)\right\} \right\}$$

(rSquaredMiddle /. {a → π / 2} /. %⟦4⟧) // FullSimplify

$$\frac{71}{54} - \frac{2\sqrt{2}}{9}$$

N[Sqrt[%] - 1, 25] (∗ Worst of middle-match errors ∗)

0.0002725300074277054901705054